Pricing Perishables

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Abstract

A key feature of food products is their perishability. Within the short marketing window that characterizes most food and ag products, demand is typically highly stochastic and difficult to predict. This combination of features poses substantial challenges to retailers when pricing products and has implications for performance that ripples through vertical food chains. For many food products, processing to forms that can be preserved and held in inventory has traditionally been used as a means of coping with these conditions, despite its high costs and ancillary risks introduced such as change in product attributes and deterioration. This paper presents an alternative ERM strategy that focuses on dynamic pricing to control the rate of sale for perishable products. The paper considers a retailer that has market power to price and supplies perishable products to a market with substitute products and demand originating from heterogeneous consumers. Perishability implies a finite horizon for the marketing of the products over which demand across market segments of consumers is both dynamic and stochastic. Faced with uncertainty, we suppose the firm has limited information about the stochastic properties of demand and must choose a pricing strategy that projects over the market horizon. This price trajectory represents a key control mechanism to cope with uncertainty of both the perishability of the product and of demand.

A variety of mechanisms for setting prices has been pursued in the past and can be imagined. Given substantial waste associated with food retailing, it seems evident that retailers may not incorporate the social or food supply network interests in optimal performance. Uniform pricing within the marketing horizon is typical for most food retailers. At the horizon, or shelf-life, the product is often removed from the shelves and either disposed of, or diverted into a secondary market. Despite such practice, lessons exist from other industries where dynamic pricing approaches have been pursued. Here, we consider pricing rules derived from robust optimization that sets price trajectories over the market horizon that explicitly consider two features that appear to be of particular interest for food: 1) sales of all available supply (i.e. eliminate disposal) and 2) existence of close substitutes (i.e. fresher product). An important innovation in many industries for dynamic is the concept of price assurance. We consider two types of price assurance. Under *ex-post* price assurance, prices are set subject to the constraint that refunds will be paid if future prices are reduced below levels paid by consumers. This is an intriguing variation on "everyday low pricing". Next, we introduce a novel alternative that we label as *ex-ante* price assurance where the firm sets the dynamic price trajectory subject to the constraint that prices will not decrease. Though perhaps counter-intuitive in perishable good context, we show this novel approach has merit under particular demand conditions. Thus, we compare three dynamic pricing strategies to manage uncertain demand given perishability: i) robust dynamic pricing, ii) *ex-post* price assurance, and iii) *ex-ante* price assurance. Numerical experiments show that our robust optimization model prevents loss when a firm encounters the worst case demand and outperforms a deterministic pricing model. Comparison across different pricing strategies identifies conditions under which particular strategies are superior to the others.

Keywords: *Dynamic pricing, demand substitution, consumer choice model, robust optimization, price assurance*.

1 Introduction

Enterprise Risk Management (ERM), focuses on integration of process control to mitigate risk. A critical source of risk faced by enterprises originates from uncertain demand. Control of the implication of such uncertainty has often focused on coping strategies such as diversification, inventory holding, or off-putting through contracting in a variety of forms. In each case, such coping incurs high costs. In this paper, we explore alternative policies that integrate inventory policy for perishable products with their pricing. Faced with demand uncertainty, we show results that confirm that the firm can benefit by adopting robust pricing strategies to manage sales of the products across heterogeneous consumers. We consider robust dynamic pricing and extend it to incorporate ex-post and ex-ante price assurance thereby introducing new tools for applying ERM to marketing operations.

The possibility of using price as a control of sales for perishable products has been widely recognized as useful. However, where substitutes exist, such dynamic price policy must be constrained to recognize the potential for product switching. Further, perishability suggests the firm faces a constraint to sell all available inventories. When opening season supply is pre-determined by earlier time intensive production, the absence of inventory replenishment over the marketing season implies prices will likely vary. It has been traditional that in this case prices decrease to ensure inventory clearance. When unanticipated demand shocks threaten to reduce sales, substantial price decreases may be pursued. If consumers are strategic and anticipate the possibility of price decreases, purchases may be postponed shifting sales into the future and rendering season sales more uncertain. In this context, ERM strategies for managing the marketing of perishables are of great interest and are the subject of this paper. Here we consider dynamic pricing strategies to mitigate demand uncertainty and show that strategies based on robust optimization admitting uncertainty offer an important alternative to dynamic pricing based on point forecasts of demand. We consider robust dynamic pricing first and then consider price assurance policies that seek to mitigate the possibility that consumers postpone purchases resulting in expanded risk. Two types of price assurance are considered. Under *ex-post* price assurance prices are set subject to the constraint that refunds will be paid if future prices are reduced below levels paid by consumers. Next, we introduce a novel alternative we label as *ex-ante* price assurance where the firm sets the dynamic price trajectory subject to the constraint that prices will not decrease. Though perhaps counter-intuitive in perishable good context, we show this novel approach has merit under particular demand conditions.

The plan of the paper is as follows. Section 2 reviews relevant literature regarding perishable product pricing, customer behavior, price assurance, as well as robust optimization. Section 3 provides a robust dynamic pricing model and then compares the robust policy with a dynamic pricing policy based on point forecasts. Section 4 provides robust models for price assurance policy (ex-post and ex-ante). Section 5 presents numerical experiment results and compares the revenue implications of the alternative dynamic pricing models. Section 6 provides conclusions.

2 Literature Review

The revenue management and dynamic pricing problem for perishable products has considered dynamic pricing, inventory control or a capacity control, for dominant firms with monopolistic pricing power. The models have been developed in a certain environment such as a fixed-inventory, no-replenishment, perishable multi-products, price-sensitive demand and price-sensitive customers (see Elmaghraby and Keskinocak 2003, Bitran and Caldentey 2003, Talluri and Van Ryzin 2005).

The implications of substitute goods for dynamic pricing have been considered by Bitran et al. 2006 using a consumer choice model that incorporates budget constraints. Gallego et al. 2004 and Liu and van Ryzin 2008 proposed a choice-based deterministic linear programming (CDLP) model using multinomial logit model (MNL) to describe customer strategic behavior. Zhang and Cooper 2005 proposed an approximation of the dynamic pricing for the multi-substitutable-flight problem and provided bounds to the optimal revenue. Zhang and Cooper 2009 extend this earlier work using a discrete-time Markov decision process model. Dong et al. 2009 present a dynamic pricing model where customer's behavior has been formulated in a multinomial logit model and consider the implications of stock-out risk. Su 2007 introduce strategic consumer behavior in an intertemporal pricing model which considers customers' different valuations and different degrees of patience to show that strategic waiting by customers is sometimes beneficial to firms. Su and Zhang 2008 consider a similar specification within the context of supply chain performance under demand uncertainty. Aviv and Pazgal 2008 consider impatient and patient customers in a dynamic pricing context. Levin et al. 2009 show results further suggest the importance of considering customer strategic behavior.

Levin et al. 2007 present a price assurance model in a monopolistic setting where customers would be reimbursed for the price difference between a purchase time and a future time. In this paper, customers are partitioned into three segments: 1) do not buy, 2) buy without and 3) buy with a price guarantee option. These studies are based on the concept of the option in finance to mitigate the risk by future price uncertainty that customers feel. Purchase of the option by paying the extra payment can reduce consumers' anxiety and enhance revenues. Lai et al. 2009 studied the impacts of posterior price assurance policy considering strategic consumers under total demand uncertainty. They found that the policy eliminates strategic consumers' action (buy now or wait).

The majority of past literature has developed deterministic models, while a limited segment has considered demand to be characterized by known distributions. Importantly, we need to notice that solutions to optimization problems can be remarkably sensitive to how uncertainty is characterized (Bertsimas and Sim 2004). Under risk neutrality, the approach of using expected values fails to reflect the worst case scenarios or is difficult to use when information concerning underlying distributions is limited. Therefore, within a deterministic framework, Soyster 1973 proposed initially the concept of a robust optimization considering simple perturbations on parameters in a linear optimization system. The idea behind *robust optimization* is to consider the worst case scenario without a specific distribution assumption. Following this work, Ben-Tal and Nemirovski 1999 and Bertsimas and Sim 2003, Ben-Tal et al. 2004 presented models that can adjust for loss aversion by defining different uncertainty sets such as an ellipsoidal, polyhedral and cardinality set. The robust optimization method has been widely used in many different applications such as finance and discrete event problems such as the evacuation problem. Efforts to determine robust policies in revenue management area have recently been highlighted by Lan et al. 2008 and Birbil et al. 2009. Their studies develop robust models for a single leg problem and compares to other traditional pricing models. However, these models for robust policy fail to consider substitutes, customer behavior, or price assurance policy.

3 Dynamic pricing strategies

In this section, we derive and compare optimal dynamic pricing from a revenue management models for substitutable, perishable products based two approaches to considering stochastic over a finite marketing season. In the first, we consider a model consistent with pricing based on point forecasts of demand. In the second, we explicitly acknowledge uncertainty that characterizes demand and suppose the decision maker derives a robust dynamic pricing policy. Our results highlight the potential benefits of strategies to manage under uncertain demand using robust optimization.

Consider a supplier that coordinates production of *J* perishable products marketable within a finite season with dates $t = 0,1,...T$. Demand is heterogeneous and is characterized by *S* consumer segments with each segment noted as $s \in S_0 = \{1,2,..., S\}$ where the value of *s* is interpreted as indicating market segment such that as *s* increases, the quality and therefore the price products increases. We define demand for each market segment as $\delta_j^s(t) \in \{\delta_j^1(t), \delta_j^2(t),....., \delta_j^S(t)\}$ for product *j* for $j = 1,..., J$. We consider only the marketing problem and suppose initial stocks are pre-determined by prior production decisions. At any time *t* in the season, the supplier offers a *1 x J* vector of fixed supplies $q_s(t) = (q_s^1(t),..., q_s^J(t))$ to each segment s that represents remaining inventory given initial stocks of $q_s^j(0)$. Thus, operationally the firm is faced with sunk costs for an inventory that must be sold before the end of the season. Here, we suppose the control available to achieve this goal is the intertemporal price policy. That is, at each time *t* the firm chooses a price vector incorporating a price $p_s^j(t)$ for each product *j* for $j = 1,..., J$ and for each market segment *s* for $s = 1,..., S$ to maximize revenue.

3.1 Forecast-based, deterministic pricing model for substitutable products

We first consider the case where the firm forecasts demand and accepts those forecasts as its expectations. Where the firm's objective is linear in the stochastic factors, such an approach is equivalent one where the firm is risk neutral, having preferences only for the first moment of a known distribution over the stochastic factor. Given its dominance in the market, the firm seeks a pricing strategy to maximize revenue by controlling inventories. Product substitutability implies that demand for a product *j* depends on the price for a product *j* and the other products $-j$. We specify demand as follows:

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k\in J, k\neq j} \gamma_s^k(t)p_s^k(t), \ \forall s, j, t$$

Our specification considers aggregate demand for segment *s* such that discrete choices by consumers across the product set define a continuous function in prices. We suppose demand is is a negative monotonic function of own price *j* and substitutes *–j*. See Maglaras and Meissner 2006 or Perakis and Sood 2006 for similar specifications. We interpret $a_s^j(t) \in \Re_+^{S\times J\times(T+1)}$ as the market potential (i.e. maximum quantitative scale) for product *j* and segment *s* at time *t*. The parameters $\beta_s^j(t) \in \Re_+^{S\times J\times(T+1)}$ and $\gamma_s^j(t) \in \Re_+^{S\times J\times(T+1)}$ represent price sensitivity of products *j* and *–j*, respectively. Note that our specification assumes consumers prefer to substitute products available in their segment rather than downgrading or upgrading to other segments. This is consistent with loyalty to branded versus private label products, particular classes of air line or entertainment tickets, or particular retail stores. We incorporate this specification by requiring that products are differentiated by market segment such that each product type *j* will be differentiated by market segment *s* such that its price will increase with *s*, see Birbil, et al. 2009 for a similar specification.

The firm's pricing problem is complicated by strategic behavior by consumers. That is, we suppose that consumers anticipate that the firm will price goods to drive end of season stocks to a minimum. Within this context, as noted by Su and Zhang 2008 and Lai, et al. 2009, strategic customers will postpone purchases to access future prices. We define the proportion of strategic customers as $\eta_s^j(t) \in \Re_+^{S\times J\times(T+1)}$. From the firm's perspective, the behavior of strategic customers is to shift demand to future periods. Define the cumulative demand function as:

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))) = (1-\eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))],\ t = 1...T,\ \forall s, j$$

In the absence of replenishment, the firm controls available inventory by setting price to maximize the season's revenue according to the following problem:

$$R(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t),\forall j,s,t} \sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=0}^{T}[p_s^j(t)\tilde{\delta}_s^j(p_s^j(t);t))] \quad (1)$$

s.t. $q_s^j(t) = q_s^j(t-1) - \tilde{\delta}_s^j(p_s^j(t), p_s^j(t))),\ t = 1...T,\ \forall s, \forall j$ (2)

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t))) = (1-\eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))],\ t = 1...T,\ \forall s, j \quad (3)$$

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k\in J, k\neq j} \gamma_s^k(t)p_s^k(t), \ \forall s, j, t \quad (4)$$

$$q^j(t) \geq 0, \quad \forall t, j \quad (5)$$

$$q^j(0) = q_0^j, \quad \forall j \quad (6)$$

$$p_s^j(t) \geq 0, \qquad \forall t, j, s \tag{7}$$

$$p_s^j(t) \geq p_{s'}^j(t), \quad s > s' \in S_0 \tag{8}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}) \geq 0 \tag{9}$$

$$\tilde{\delta}_s^j(p_s^j(0), p_s^{-j}(0)) = \delta_s^j(p_s^j(0), p_s^{-j}(0)), \ \forall s, j \tag{10}$$

3.2 Optimal control under uncertainty with robust optimization

By definition, uncertainty describes conditions when a firm anticipates stochastic factors will affect its performance, however, the firm has limited knowledge of the mechanism generating the stochastic outcomes. In contrast, full knowledge of such mechanisms is presumed when a stochastic environment is described as posing risk. Given knowledge of characterizing moments of data generating mechanism, a natural approach to decision-making under risk is to suppose decision-makers have preferences over such moments and set controls to optimize a functional representation of those preferences. In sharp contrast, under uncertainty, knowledge of the shape of the distribution of stochastic factors or its moments, are not assumed. Here, we propose use of robust optimization to set performance controls in uncertain decision environments. Robust optimization for a single control problem has been recently presented by Lan, et al. 2008 and Birbil, et al. 2009. Our specification considers robust pricing across a set of substitutable products where demand across a spectrum of heterogeneous customer segments is uncertain. Our approach builds on Soyster 1973 and Ben-Tal and Nemirovski 1999.

Define the uncertainty set U_d for each demand as

$a_s^j(t) \in [\bar{a}_s^j(t)(1-\theta_s^j(t)), \bar{a}_s^j(t)(1+\theta_s^j(t))]$ where $\bar{a}_s^j(t) \in \Re^{J \times S \times (T+1)}, \bar{\theta}_s^j(t) \in \Re^{J \times S \times (T+1)}$.

Alternative specifications such as ellipsoidal and polyhedral uncertainty sets are considered by Ben-Tal and Nemirovski 1999 and Bertsimas and Sim 2004. For a particular uncertainty set, suppose an optimal price policy is derived from the revenue maximizing problem describe above. It follows that the control problem has infinite number of constraints that describe possible uncertainty sets. Since given direct solution of such a problem is intractable, we manipulate the specification to transform the control problem to an equivalent form. Specifically, we propose the following deterministic problem as equivalent to the robust formulation with uncertain demand (see appendix A for derivation):

$$R(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t), \forall j,s,t,V} V \tag{11}$$

s.t.

$$\begin{aligned}\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \Bigg[p_s^j(t) \Bigg\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \\ + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \Bigg\} \Bigg] \geq V\end{aligned} \tag{12}$$

$$\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \geq 0 \tag{13}$$

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \right.$$
$$\left. + \sum_{\tau=1}^{i} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau)) - \beta_s^j(\tau) p_s^j(\tau) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(\tau) p_s^k(\tau) \right) \right) \geq 0, \forall s, j, t \tag{14}$$

$$p_s^j(t) \geq 0, \qquad \forall t, j, s \tag{15}$$

$$p_s^j(t) \geq p_{s'}^j(t), \quad s > s' \in S_0 \tag{16}$$

3.3 Evaluation of robust, dynamic pricing policy

In this section, we compare the robust price policy against the policy by a forecast-based model. First, we present optimal price policies and inventories for each models derived from a numerical example. Next, we consider how the robust policy varies as the extent of uncertainty varies across a set of randomly generated scenarios.

In our numerical experiments, we assume that consumer segment 1 focuses on the highest quality and priced products. Parameters for numerical experiments are given in Table 1. We limit our consideration to demand parameters satisfying Definition 1.

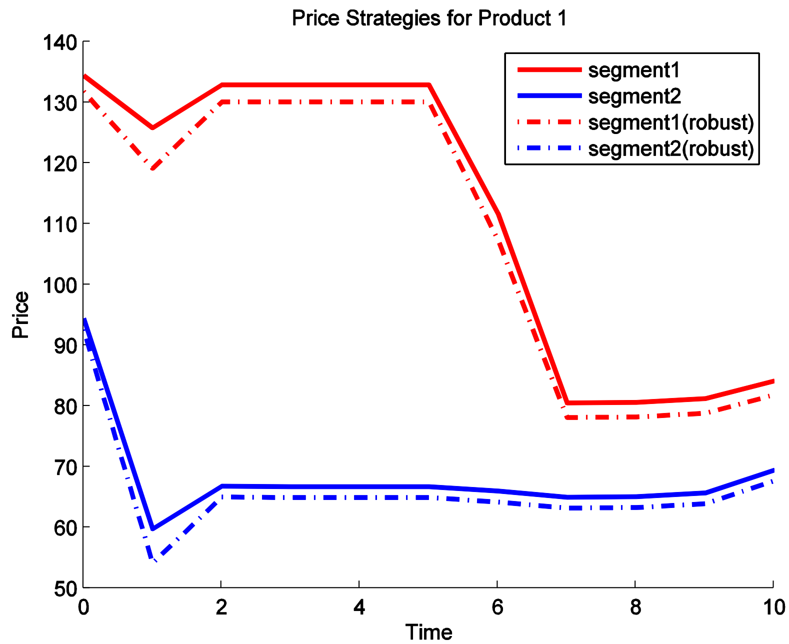
Definition 1. Customer segment s is of a rank that is higher than that of segment s' if $a_s^j(t) \leq a_{s'}^j(t)$, $\beta_s^j(t) \leq \beta_{s'}^j(t)$, and $\gamma_s^j(t) \leq \gamma_{s'}^j(t)$ for $\forall j, t$.

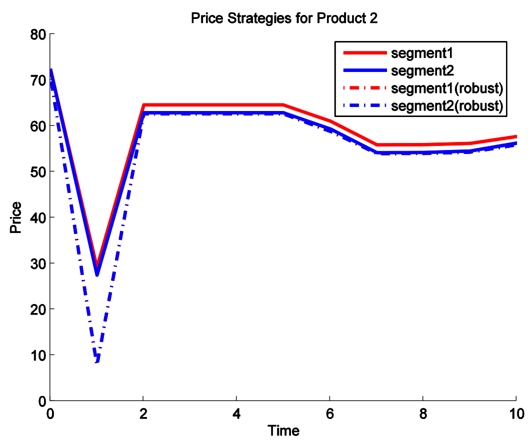
We assume demand parameters satisfy Definition 1. This implies as *s* decreases, segment rank increases, and market potential and price sensitivity decreases.

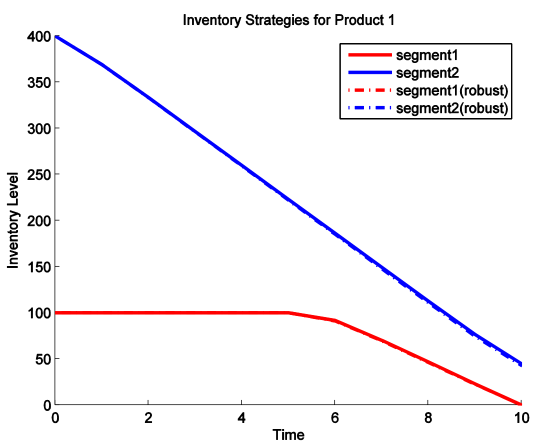
Table 1. Parameters for $j=2$, $s=2$, $T=10$

Parameters	Product 1		Product 2	
	Segment 1	Segment 2	Segment 1	Segment 2
$a_s^j(t)$	60	120	30	100
$\beta_s^j(t)$	0.5	1.5	0.6	1.6
$\gamma_s^j(t)$	0.1	0.3	0.1	0.3
$\eta_s^j(t)$	0.2	0.2	0.2	0.2
$\omega_s(t)$	0.5	0.5	0.5	0.5
$\theta_s^j(t)$	0.02	0.02	0.02	0.02
$q^j(0)$	100	400	80	300

The experiments are implemented using MATLAB and GAMS on a machine with Windows XP OS, T2300 CPU, 1.66 GHz, and 1 GB RAM. The optimal price strategies and inventory policies are shown in Figure 1. In the figure, solid and dotted lines represent optimal strategy based on forecasts and robust strategy based on uncertain demand, respectively. Red lines and blue lines are the strategies for segment 1 and 2, respectively.


Price Strategies for Product 1
Price
Time
segment1
segment2
segment1(robust)
segment2(robust)
140
130
120
110
100
90
80
70
60
50
0
2
4
6
8
10


Price Strategies for Product 2
Price
Time
segment1
segment2
segment1(robust)
segment2(robust)
80
70
60
50
40
30
20
10
0
0
2
4
6
8
10


Inventory Strategies for Product 1
Inventory Level
Time
segment1
segment2
segment1(robust)
segment2(robust)
400
350
300
250
200
150
100
50
0
0
2
4
6
8
10

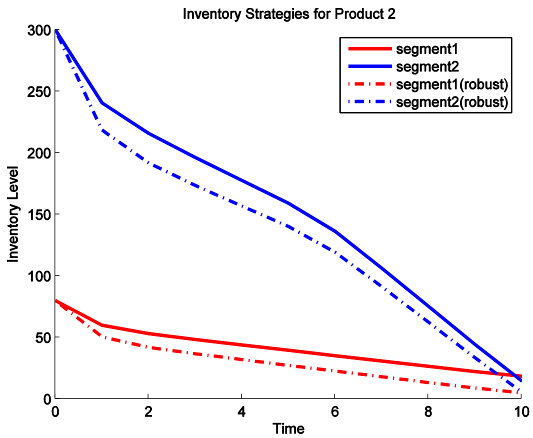


Figure 1. Dynamic price and inventory strategies (a deterministic and a robust model)

Results in Figure 1 show that optimal price strategy under uncertainty sets prices lower than one based on forecasts. This is consistent with aversion to future uncertainty that leads to pricing that induces greater sales early in the season to eliminate possible lost sales. Looking across market segments, we find the price for segment 1 is much higher than the price for segment 2 reflecting the economic benefits of differentiation. A similar price difference across segments is found under both demand conditions. For the inventories, we can see that robust strategies for product 1 are very similar to forecast-based ones, while the inventory trajectories for product 2 are very between to two market segments. We also see that for product 2, inventory is reduced under a robust strategy relative to forecast-based strategy.

Next, we consider sensitivity of firm revenue to the extent or level of uncertainty. We consider the set of values for $\theta_s^j(t)$ as [2%, 5%, 7%, 10%, 15%] . This results in uncertainty defined by the limits of possible values increasing by 4%, 10%, 14%, 20% and 30%, respectively. Thus, for the uncertainty case, we derive five robust price strategy trajectories that can be compared to the forecast-based strategy. Based on derived price trajectories, we generate random demand sets from a uniform distribution to generate demand realizations that define a set of scenarios. We draw our levels of market potential from the interval $[\bar{a}_s^j(t)(1-\theta_s^j(t)), \bar{a}_s^j(t)(1+\theta_s^j(t))]$. We derive 50 realizations (scenarios) for each setting. For each scenario, we derive corresponding revenue trajectories based on optimal dynamic price trajectories. To compare results, we note the forecast-based trajectories (*D*) and robust trajectories (*R*) and, for each scenario, consider optimal value (*ov*), as well as minimum (*min*), maximum (*max*), average (*ave*), and standard deviation (*sd*) of objective values. Results are presented as in Table 2.

Table 2. Simulation experiment under randomly generated scenarios

θ	D_{ov}	R_{ov}	$D_{\max}$	$R_{\max}$	$D_{\min}$	$R_{\min}$	D_{ave}	R_{ave}	D_{sd}	R_{sd}
0.02	4.9712e+004	4.6854e+004	5.0784e+004	5.1757e+004	4.9178e+004	4.9876e+004	4.9861e+004	5.0743e+004	359.5613	331.9319
0.05	4.9712e+004	4.4984e+004	5.2824e+004	5.7034e+004	4.8641e+004	5.2732e+004	5.0463e+004	5.4621e+004	944.2957	910.4214
0.07	4.9712e+004	4.3783e+004	5.3800e+004	6.0095e+004	4.6941e+004	5.3378e+004	5.0089e+004	5.6313e+004	1.5416e+003	1.4640e+003
0.1	4.9712e+004	4.1946e+004	5.4003e+004	6.2711e+004	4.7689e+004	5.6926e+004	5.0485e+004	5.9412e+004	1.6425e+003	1.4787e+003
0.15	4.9712e+004	3.7643e+004	5.5573e+004	6.5213e+004	4.5801e+004	5.7144e+004	5.1140e+004	6.1633e+004	2.6568e+003	2.2816e+003

From the experiment, robust optimization solutions are more stable (based on standard deviation) than forecast-based solutions. This result supports the recommendation that adoption of robust price strategies be pursued relative to forecast-based strategies to stabilize revenue. That is, the robust strategy enables the firm to cope with sudden demand decreases as the range of performance that would results is smaller than that under forecast-based strategy. The same recommendation to adopt robust dynamic pricing is supported by considering minimum, maximum, and average revenues achievable. When we compare the optimal values D_{ov} and R_{ov}, the values of R_{ov} are decreasing in the amount of uncertainty and smaller than D_{ov}. But, for random scenarios, the robust model outperforms. This implies that forecast-based strategies derived from our model may result in reduced sales relative to robust strategies.

4 Price assurance pricing models

Thus far, we have considered the performance of a robust dynamic pricing strategy when a firm faces uncertain demand. Relative to a forecast-based strategy, we have shown the proposed robust dynamic pricing strategy enhances both the level and variation of revenues. In this section, we extend our consideration of robust optimization methods to consider an alternative to revenue control from an optimal price policy. In particular, here we acknowledge that the firm may face both demand uncertainty as well as strategic behavior by consumers. In the latter case, price variation may be anticipated by consumers and result in postponement of purchases by strategic consumers. By designing the price policy to accommodate this possibility we show the firm's performance can be further enhanced. Two price assurance policy types are considered, *ex-post* and *ex-ante*. The former is a currently used pricing method which sets price first and then offers a refund to consumers if the price falls below their purchase price. The later is an approach that is new to the literature and incorporates constraints in the price policy derivation that ensure price will not decline sufficiently to induce refunds. Before developing these pricing models, we add an assumption to characterize strategic customers.

Assumption 1. The proportion of strategic customers' waiting under pricing assurance policy, $\eta_s^j(t)$ is smaller than that under a forecast-based pricing policy.

This assumption can be justified by Lai, et al. 2009 who find that an ex-post price assurance policy eliminates strategic consumers' action (buy now or wait).

4.1 Ex-post price assurance pricing model

When the price assurance policy is offered, the firm is obliged to refund the price difference between each purchase price at time t ($p_s^j(t)$) and lowest prices until a final time ($\min_{\tau>t} p_s^j(\tau)$). Strategic customers amounting to a proportion $\omega_s(t) \in \Re^{S\times T}$ will claim a refund. Thus, the penalty associated with a price assurance policy can be expressed as follows:

$$\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=0}^{T-1}(p_s^j(t)-\min_{\forall\tau>t}p_s^j(\tau))^+\omega_s(t)\tilde{\delta}_s^j(t),$$

where $X^+ = \max(X,0)$.

Incorporating this penalty function, we have a robust price assurance model as follows:

$$R_{post}(p_s^j(t);I_s^j(t)) = \max_{p_s^j(t),\forall j,s,t,V} V \tag{17}$$

s.t.

$$\begin{aligned}&\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=0}^{T}\Bigg[p_s^j(t)\Bigg\{\left(\left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0)\right.\\
&\quad\left.+\sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\left(\bar{a}_s^j(t)(1-\theta_s^j(t))-\beta_s^j(t)p_s^j(t)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(t)p_s^k(t)\right)\right)\Bigg\}\Bigg]\\
&\quad-\sum_{j=1}^{J}\sum_{s=1}^{S}\sum_{t=0}^{T-1}(p_s^j(t)-\min_{\tau>t}p_s^j(\tau))^+\omega_s(\tau)\\
&\quad\times\left(\left\{\left(\left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0)\right.\right.\right.\\
&\quad\left.\left.\left.++\sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\left(\bar{a}_s^j(t)(1-\theta_s^j(t))-\beta_s^j(t)p_s^j(t)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(t)p_s^k(t)\right)\right)\right\}\right)\geq V\end{aligned} \tag{18}$$

$$\bar{a}_s^j(t)(1-\theta_s^j(t))-\beta_s^j(t)p_s^j(t)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(t)p_s^k(t)\geq 0 \tag{19}$$

$$\begin{aligned}q_s^j(t) &= q_s^j(0)-\sum_{i=0}^{t-1}\left(\left(\prod_{\tau=1}^{i}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0)\right.\\
&\left.+\sum_{\tau=1}^{i}(1-\eta_s^j(\tau))^{t-\tau}\left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau))-\beta_s^j(\tau)p_s^j(\tau)+\sum_{\forall k\in J,k\neq j}\gamma_s^k(\tau)p_s^k(\tau)\right)\right)\geq 0,\forall s,j,t\end{aligned} \tag{20}$$

$$p_s^j(t)\geq 0,\qquad \forall t,j,s \tag{21}$$

$$p_s^j(t)\geq p_{s'}^j(t),\quad s>s'\in S_0 \tag{22}$$

From this notation, it is clear that price assurance policy implies penalties will be paid, however, it is also apparent that by optimal design of the price policy the number of strategic customers waiting can be reduced.

4.2 Ex-ante pricing model under price assurance

As an alternative pricing method over a price assurance policy, a firm can choose to set prices pre-empt the possibility of refunds by simply choosing a trajectory such that prices never decrease. This pricing scheme can be easily formulated by introducing the following condition.

$p_s^j(t) \ge p_s^j(t-1)$ for $t = 1,\dots,T$.

While intuitive with respect to discouraging strategic behavior by consumers, this policy would seem counter-intuitive relative to the need to clear inventories. For this scheme of pricing, by incorporating robust optimization, we have:

$$R_{ante}(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t), \forall j,s,t,V} V \tag{23}$$

s.t.

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \right\} \right] \ge V \tag{24}$$

$$\bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \ge 0 \tag{25}$$

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1} \left(\left(\prod_{\tau=1}^{i} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{i} (1-\eta_s^j(\tau))^{t-\tau} \left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau)) - \beta_s^j(\tau) p_s^j(\tau) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(\tau) p_s^k(\tau) \right) \right) \ge 0, \forall s, j, t \tag{26}$$

$$p_s^j(t) \ge 0, \qquad \forall t, j, s \tag{27}$$

$$p_s^j(t) \ge p_{s'}^j(t), \quad s > s' \in S_0 \tag{28}$$

$$p_s^j(t+1) \ge p_s^j(t) \tag{29}$$

5 Comparison of dynamic pricing strategies under demand uncertainty

Thus far, we have developed i) a robust model without considering price assurance and ii) a robust ex-post and iii) a robust ex-post model. In this section, we will investigate conditions under which one model is superior the others by numerical experiments.

5.1 Impacts of different customer behaviors

A key element in our specification is the behavior of customers under each pricing strategy. Thus, in this subsection, we study and compare three different pricing models with respect to variation in customer behavior.

Let R_{cont}, R_{post}, R_{ante} be the revenue associated with robust, an ex-post, and an ex-ante pricing strategies. Here, we know that $\eta_s^j(t)$ and $w_s(t)$ are important factors to differentiate those pricing schemes and to represent the customers' behavior. How many customers would wait strategically might differ depending on whether a firm would offer price assurance policy or not. Also, how many customers would claim refunds does not matter to a firm if it would adopt an ex-ante model. For this reason, we compare the solutions for different parameter values of $\eta_s^j(t)$ and $w_s(t)$. The experiments are conducted for parameters in Table 1 and the results are shown in Table 3.

Table 3. Simulation experiments for different customer behavior

$\eta_s^j(t)$	0		0.1		0.2		0.2
$w_s(t)$	R_{post}	R_{ante}	R_{post}	R_{ante}	R_{post}	R_{ante}	R_{cont}
0.3	5.9556e+004	6.3480e+004	4.8749e+004	8.5010e+003	4.6388e+004	8.1104e+003	4.6854e+004
0.5	5.8568e+004	6.3480e+004	4.8453e+004	8.5010e+003	4.6077e+004	8.1104e+003	4.6854e+004
0.7	5.7059e+004	6.3480e+004	4.8158e+004	8.5010e+003	4.5766e+004	8.1104e+003	4.6854e+004
0.9	5.6251e+004	6.3480e+004	4.7862e+004	8.5010e+003	4.5456e+004	8.1104e+003	4.6854e+004

As shown in Table 3, an ex-post and an ex-ante pricing model incurs smaller revenues than a robust pricing model, when the proportion of strategically-waiting customers is the same $\eta_s^j(t)$=0.2. This result is quite intuitive because a firm should pay refunds to customers or avoid the refunds. However, as mentioned in Assumption 1, addressing a pricing assurance policy can reduce customers' strategic waiting actions (Lai, et al. 2009). Thus, when we compare solutions of the price assurance models for $\eta_s^j(t)$=0 to the robust model for $\eta_s^j(t)$=0.2, the price assurance model is superior to the robust model. Between two pricing models for price assurance, the ex-ante model outperforms the ex-post model, for $\eta_s^j(t)$=0 and the ex-post model is better for $\eta_s^j(t)$=0.1, respectively. Thus, if a firm can eliminate the customers' strategic waiting significantly, the ex-ante pricing model performs best.

Moreover, consistent with our intuition, we can see that the revenues are generally decreasing when the proportion of customers' claims increases. However, the rate of reduction is not significant, compared to the change of $\eta_s^j(t)$ implying that the proportion of waiting is more critical than the proportion of claim.

5.2 Time-variant demand

Numerical experiments so far were conducted for time-invariant demands. However, in the real world, a firm might have increasing or decreasing demand over time. Thus, in this subsection, we investigate how revenues change for increasing or decreasing demand. First, we consider the case of increasing demand, in other words,

$$a_s^j(t) > a_s^j(t+1),\ \forall t = 1,\ldots,T$$

Then, we examine which policy would be more beneficial, when demand decreases. For numerical experiments, we assume parameters as follows and keep other parameters the same as in Table 1:

$a_1^1(t) = 60 - kt$, $a_2^1(t) = 120 - kt$, $a_1^2(t) = 30 - kt$, and $a_2^2(t) = 100 - kt$, for $k \in \Re_{++}$.

$\eta_s^j(t)$=0.2 for a robust model,

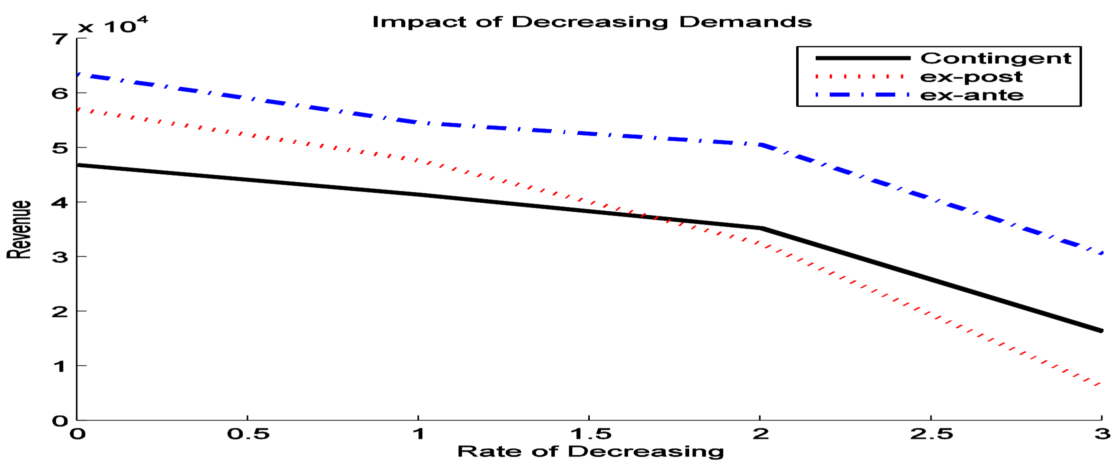
$\eta_s^j(t)$=0 for an ex-ante and an ex-post model

$w_s(t)$=0.7 for an ex-ante model

The results for decreasing demands are described in Table 4 and Figure 2(a). Like our intuition, the revenues for all pricing models are decreasing. As shown in Figure (a), revenues for all pricing models are decreasing steadily for smaller changing rate (*k*) and then drastically for higher *k*. However, more important results are the rate of decreasing revenue, which value is in parenthesis in Table 4. The rate of ex-ante pricing model is smaller (between 7% and 40%) than others and the rate of ex-post pricing model is very sensitive to the change of decreasing demands (between 16% and 80%). Thus, when a firm faces a decreasing demand, ex-ante pricing model can incur more stable revenues.

Table 4. Simulation experiments for different customer behaviors

$\eta_s^j(t)$	0		0.1		0.2		0.2
$w_s(t)$	R_{post}	R_{ante}	R_{post}	R_{ante}	R_{post}	R_{ante}	R_{cont}
0.3	5.9556e+004	6.3480e+004	4.8749e+004	8.5010e+003	4.6388e+004	8.1104e+003	4.6854e+004
0.5	5.8568e+004	6.3480e+004	4.8453e+004	8.5010e+003	4.6077e+004	8.1104e+003	4.6854e+004
0.7	5.7059e+004	6.3480e+004	4.8158e+004	8.5010e+003	4.5766e+004	8.1104e+003	4.6854e+004
0.9	5.6251e+004	6.3480e+004	4.7862e+004	8.5010e+003	4.5456e+004	8.1104e+003	4.6854e+004



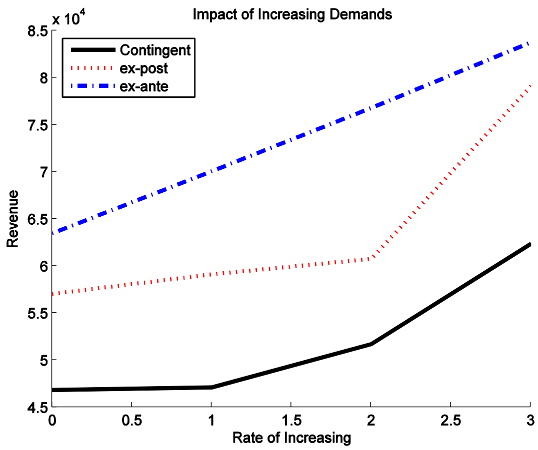


Figure 2. The impact of time-varying demand on different pricing models (a) decreasing demand (left), (b) increasing demand (right)

Similarly, we will investigate the impact of increasing total demand over time and compare three models. To express increasing demand over time, we assume parameters as follows:

$a_1^1(t) = 60 + kt$, $a_2^1(t) = 120 + kt$, $a_1^2(t) = 30 + kt$, and $a_2^2(t) = 100 + kt$ for $k \in \Re_{++}$.

The results can be found in Table 5 and Figure 2(b). Similarly to the case of decreasing demands, the ex-ante model is most stable (rates are around 10%) and ex-post is most sensitive changing up to 30.2%. However, differently from the decreasing demands, if a firm use the ex-post pricing scheme appropriately, it can accomplish more revenues.

Table 5. Simulation experiments for increasing demands (values in parenthesis represents rate of increment)

k	R_{con}	R_{post}	R_{ante}
0	4.6854e+004	5.7059e+004	6.3480e+004
1	4.7128e+004 (0.6%)	5.9136e+004 (3.6%)	7.0074e+004 (10.4%)
2	5.1732e+004 (9.8%)	6.0781e+004 (2.8%)	7.6802e+004 (9.6%)
3	6.2379e+004 (20.6%)	7.9116e+004 (30.2%)	8.3775e+004 (9.1%)

6 Conclusions and further studies

In this paper, we have presented three robust pricing models: i) a robust optimization with uncertainty without consideration of price assurance ii) ex-post price assurance, and iii) ex-ante price assurance. By applying a robust optimization method, we can have robust dynamic pricing policies under demand uncertainty which guarantee a certain level of performance for possible scenarios, even for the worst case scenario. In other words, our robust models can prevent a risk that demand falls significantly. Then, we compared different pricing schemes, especially focusing on the price assurance policy under which a firm can reduce customers' strategic actions such as waiting for a future cheaper price.

Our experiments for these proposed pricing models provides us interesting implication that price strategies under a robust model are very stable for many different randomly generated scenarios. Even if a firm encounters sudden demand fall, a certain range of performance would be guaranteed and the robust policy can incur more revenues than a policy given by a deterministic model. Also, we have studied the impact of customer's behavior such as how many customers wait for a future discounted price and claim the refund under price assurance policy. Our results show that price assurance policy outperforms a robust pricing model, when it can eliminate or significantly reduce the customer's strategic waiting. Also, we can see that the proportion of waiting is more important than the proportion of claim. Besides, when a firm faces a decreasing or increasing demand, the ex-ante pricing model is the least sensitive to revenues, the robust is intermediate, and the ex-post model is highly sensitive. Therefore, as demand is expected to decrease over time, the ex-ante price assurance model could be a good scheme that a firm can take. For the opposite case, the ex-post price assurance model could make more revenue.

In this paper, even though we propose robust pricing models, from the robust optimization modeling perspective, we can improve current demand uncertainty set. As discussed in Bertsimas and Sim 2004, different definitions of uncertainty set can make the models less conservative and more realistic. Also, uncertainty in other parameters can be another extension. For example, a firm might want to have a robust policy against customer's behavior.

APPENDIX. Derivation of Robust Optimization model

By addressing uncertainty set (A.12) and manipulating objective function, we have

$$R(p_s^j(t); I_s^j(t)) = \max_{p_s^j(t),\forall j,s,t} V \tag{A.1}$$

$$\text{s.t.} \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} [p_s^j(t)\tilde{\delta}_s^j(p_s^j(t);t))] \geq V \tag{A.2}$$

$$q_s^j(t) = q_s^j(t-1) - \tilde{\delta}_s^j(p_s^j(t), p_s^j(t))), \quad t = 1...T, \ \forall s, \forall j \tag{A.3}$$

$$\tilde{\delta}_s^j(p_s^j(t)) = (1-\eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))], \ t = 1,...,T, \ \forall j \tag{A.4}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k\in J, k\neq j} \gamma_s^k(t)p_s^k(t), \ \forall s, j, t \tag{A.5}$$

$$q^j(t) \geq 0, \quad \forall t, j \tag{A.6}$$

$$q^j(0) = q_0^j, \quad \forall j \tag{A.7}$$

$$p_s^j(t) \geq 0, \quad \forall t, j, s \tag{A.8}$$

$$p_s^j(t) \geq p_{s'}^j(t), \quad s > s' \in S_0 \tag{A.9}$$

$$\delta_s^j(p_s^j(t), p_s^{-j}) \geq 0 \tag{A.10}$$

$$\tilde{\delta}_s^j(p_s^j(0), p_s^{-j}(0)) = \delta_s^j(p_s^j(0), p_s^{-j}(0)), \ \forall s, j \tag{A.11}$$

$$\forall a_s^j \in [\bar{a}_s^j(1-\theta_s^j), \bar{a}_s^j(1+\theta_s^j)], \ \forall t, j, s \tag{A.12}$$

Note that the constraint (A.12) makes infinite number of constraints for the revenue maximizing problem. Thus, we need to manipulate this constraint.

Here, we know that constraint (A.5) over (A.12) can be reformulated as follows:

$$\delta_s^j(p_s^j(t), p_s^{-j}(t)) = a_s^j(t) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k\in J, k\neq j} \gamma_s^k(t)p_s^k(t)$$

$$\geq \bar{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t)p_s^j(t) + \sum_{\forall k\in J, k\neq j} \gamma_s^k(t)p_s^k(t) \geq 0$$

Also, manipulation of (A.11) and (A.4) gives us:

$$\tilde{\delta}_s^j(p_s^j(t), p_s^{-j}(t)) = (1-\eta_s^j(t))[\eta_s^j(t-1)\tilde{\delta}_s^j(t-1) + \delta_s^j(p_s^j(t), p_s^{-j}(t))]$$

$$= \left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0) + \sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\delta_s^j(p_s^j(\tau), p_s^{-j}(\tau))$$

The constraint (A.6) and the above equation becomes

$$q_s^j(t) = q_s^j(t-1) - \left(\left(\prod_{\tau=1}^{t}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0) + \sum_{\tau=1}^{t}(1-\eta_s^j(\tau))^{t-\tau}\delta_s^j(p_s^j(\tau), p_s^{-j}(\tau))\right) \ t = 1,...T, \ \forall j \ .$$

From the (A.7), we have (A.6) over (A.12) as follows:

$$q_s^j(t) = q_s^j(0) - \sum_{i=0}^{t-1}\left(\left(\prod_{\tau=1}^{i}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0) + \sum_{\tau=1}^{i}(1-\eta_s^j(\tau))^{t-\tau}\delta_s^j(p_s^j(\tau), p_s^{-j}(\tau))\right)$$

$$\geq q^j(0) - \sum_{i=0}^{t-1}\left(\left(\prod_{\tau=1}^{i}(1-\eta_s^j(\tau))(\eta_s^j(\tau-1))\right)\delta_s^j(0)\right.$$

$$\left. + \sum_{\tau=1}^{i}(1-\eta_s^j(\tau))^{t-\tau}\left(\bar{a}_s^j(\tau)(1+\theta_s^j(\tau)) - \beta_s^j(\tau)p_s^j(\tau) + \sum_{\forall k\in J, k\neq j}\gamma_s^k(\tau)p_s^k(\tau)\right)\right) \geq 0$$

for $\forall t, j, s$.

Also, the constraint (A.2)

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \delta_s^j(p_s^j(\tau), p_s^{-j}(\tau)) \right\} \right] \geq V$$

$$\sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{t=0}^{T} \left[p_s^j(t) \left\{ \left(\prod_{\tau=1}^{t} (1-\eta_s^j(\tau))(\eta_s^j(\tau-1)) \right) \delta_s^j(0) \right.\right.$$

$$\left.\left. + \sum_{\tau=1}^{t} (1-\eta_s^j(\tau))^{t-\tau} \left(\overline{a}_s^j(t)(1-\theta_s^j(t)) - \beta_s^j(t) p_s^j(t) + \sum_{\forall k \in J, k \neq j} \gamma_s^k(t) p_s^k(t) \right) \right\} \right] \geq V$$

Finally, we have (11)-(16).

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